

Lincoln H. Inniss

PRACTICE QUESTIONS WITH
ANSWERS IN PURE ADVANCED LEVEL
MATHEMATICS BOOK 1


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CHAPTER 1

REASONING AND LOGIC

TRUTH TABLES

NOTES:

- (1) The symbol \wedge means ‘and’.
- (2) The symbol \vee means ‘or’.
- (3) The symbol \sim means ‘not’, **this is negation.**
- (4) A **proposition** is a sentence which assumes the truth value T or false, F.
 Sometimes the number 1 is used for TRUE and 0 is used for FALSE.
- (5) If p and q are **propositions** then $p \wedge q$ means ‘ p and q ’.
- (6) If p and q are **propositions** then $p \vee q$ means ‘ p or q ’.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Complete the following truth table.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee q$	$p \vee \sim q$
T	T	T	F	F	F	T	
T	F	F	T	F	T	F	
F	T	F	T	T	F	T	
F	F	F	T	T	T	T	

Complete the following truth table.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \vee (q \vee r)$	$q \wedge r$	$p \vee (q \wedge r)$	$\sim(q \vee r)$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

NOTES:

- (1) If x and y are propositions and at least one is FALSE, then $(x \wedge y)$ is FALSE.
- (2) If x and y are propositions and at least one is TRUE, then $(x \vee y)$ is TRUE.
- (3) $z \vee (x \wedge y) = (z \vee x) \wedge (z \vee y)$.
- (4) $z \wedge (x \vee y) = (z \wedge x) \vee (z \wedge y)$.

In the table below, verify that columns (5) and (8) are equivalent.

$z \vee (x \wedge y) = (z \vee x) \wedge (z \vee y)$ [The two statements are logically equivalent.]

x	y	z	$x \wedge y$	$z \vee (x \wedge y)$	$z \vee x$	$z \vee y$	$(z \vee x) \wedge (z \vee y)$
T	T	T					
T	T	F					
T	F	T					
F	T	T					
T	F	F					
F	T	F					
F	F	T					
F	F	F					

Complete the table below to verify that $z \wedge (x \vee y)$ is logically equivalent to

$(z \wedge x) \vee (z \wedge y)$.

$$z \wedge (x \vee y) = (z \wedge x) \vee (z \wedge y)$$

x	y	z	$x \vee y$	$z \wedge (x \vee y)$	$z \wedge x$	$z \wedge y$	$(z \wedge x) \vee (z \wedge y)$
T	T	T					
T	T	F					
T	F	T					
F	T	T					
T	F	F					
F	T	F					
F	F	T					
F	F	F					

NOTES:

(1) Let p = the hypothesis and q = the conclusion.

$p \rightarrow q$ means p implies q .

(2) If the statements p and q are both true, then p implies q or $(p \rightarrow q)$ is true.

(3) If the statements p and q are both false, then p implies q or $(p \rightarrow q)$ is true.

(4) If p is true and q is false then p implies q or $(p \rightarrow q)$ is false.

(5) If p is false and q is true then p implies q or $(p \rightarrow q)$ is true. That is, it is true that the false hypothesis is false.

In other words,

Column 1	Column 2	Column 3	Column 4
p	q	$p \rightarrow q$	Column 3 states that
T	T	T	It is true that the true hypothesis is true.
T	F	F	It is false that the true hypothesis is false.
F	T	T	It is true that the false hypothesis is true.
F	F	T	It is true that the false hypothesis is false.

Complete the table below.

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	T			
T	F	F			
F	T	T			
F	F	T			

Complete the table below.

p	r	$\sim p$	$\sim r$	$\sim p \vee \sim r$	$\sim p \wedge \sim r$	$\sim(p \vee r)$	$\sim(p \wedge r)$
T	T						
T	F						
F	T						
F	F						

NOTE THAT

(1) $\sim(p \vee r) = \sim p \wedge \sim r$ and (2) $\sim(p \wedge r) = \sim p \vee \sim r$.

CHAPTER 2

THE REAL NUMBER SYSTEM,

SIGMA NOTATION,

MATHEMATICAL INDUCTION

THE REAL NUMBER SYSTEM

NOTES:

Let $a, b, c \in R$,

- (1) $a + b = c$ for all $a, b, c \in R$. The set of real numbers is ***closed*** under addition.
- (2) $a - b = c$ for all $a, b, c \in R$. The set of real numbers is ***closed*** under subtraction.
- (3) $ab = c$ for all $a, b, c \in R$. The set of real numbers is ***closed*** under multiplication.
- (4) $\frac{a}{b}$ ***is not necessarily an integer.***

Thus, the set of real numbers is ***closed*** under division.

- (5) $a + b = b + a$. ***Addition is commutative.***
- (6) $a + (b + c) = (a + b) + c$. ***Addition is associative.***

(7) $a - b \neq b - a$. **Subtraction is not commutative.**

(8) $a - (b - c) \neq (a - b) - c$. **Subtraction is not associative.**

(9) $ab = ba$. **Multiplication is commutative.**

(10) $(ab)c = a(bc)$. **Multiplication is associative.**

(11) $a + 0 = a$. **The identity element under addition is ZERO.**

(12) $a + (-a) = 0$. **The inverse element under addition is $(-a)$.**

(13) $a \times 1 = a$. **The identity element under multiplication is ONE.**

(14) $a \times \left(\frac{1}{a}\right) = 1$. **The inverse element under multiplication $\left(\frac{1}{a}\right)$.**

BINARY OPERATIONS

EXERCISE 2A

1. a) Given that $x * y = 2x - y$, calculate (i) $2 * 3$, (ii) $3 * 2$, (iii) $(3 * 2) * (-4)$

(iv) $8 * (2 * 3)$.

b) Solve the equation (i) $f * 9 = 10$, (ii) $r * (-8) = 12$.

2. a) The operation $*$ is defined by $x * y = 3x + 4y$. Calculate (i) $5 * 6$,

(ii) $6 * 5$, (iii) $3 * (6 * 5)$, (iv) $(3 * 6) * 5$, (v) $(-8) * (-9)$.

b) Solve, giving the ***exact*** solution(s) of each of the following equations.

(i) $y * 4 = 14$, (ii) $x^2 * 2 = 50$, (iii) $k^7 * (-6) = 40$.

3. a) If $r * t = \frac{r+t}{r-t}$, calculate the ***exact*** value of each of the following.

(i) $3 * 6$, (ii) $5 * (-2)$, (iii) $6 * 4$, (iv) $(-10) * (5)$, (v) $(3 * 6) * 7$, (vi) $3 * (6 * 7)$.

b) (i) Is $*$ associative? (ii) Show that $*$ is not commutative.

c) Solve each of the following equations correct to 4 significant figures.

(i) $(17x) * 5 = 9$, (ii) $3x^2 * 8 = 12$.

4. a) Given that $f * g = \frac{f + g}{4f}$, calculate each of the following in **fractional form**.

(i) $8 * 2$, (ii) $5 * 8$, (iii) $(5 * 8) * 6$, (iv) $5 * (8 * 6)$.

b) Solve, giving the **exact** solution, of each of the following equations.

(i) $6x * 1 = 49$, (ii) $5x * (-2) = 10$, (iii) $7x^3 * 200 = 8$.

5. (i) Given that $p * r = p^2 - r$, calculate a) $6 * 2$, b) $8 * (-3)$, c) $(-10) * 6$,

d) $(2 * 3) * 4$, e) $2 * (3 * 4)$.

(ii) Solve, giving the solutions correct to 2 decimal places, the equation a) $x * 7 = 10$,

b) $x^5 * 8 = 4934$, c) $3x^2 * (-17) = 960$.

6. The operation $*$ is defined as $x * y = 5x^2 - 6x - 7y^2$. Solve the equation $x * 2 = 0$,

giving your answers correct to 2 decimal places.

7. The operation $\#$ is defined as $p \# r = p^2 + r^2 - 9pr$.

a) Prove that # is *commutative*.

b) Calculate $5 \# (-3)$.

ANSWERS TO EXERCISE 2A

1 a) (i) 1 (ii) 4 (iii) 12 (iv) 15

b) (i) $f=9.5$ (ii) $r=2$

2. a) (i) 39 (ii) 38 (iii) 161 (iv) 119 (v) - 60

b) (i) $-\frac{2}{3}$ (ii) $\pm\sqrt{14}$ (iii) $\sqrt[7]{\left(\frac{64}{3}\right)}$

3. a) (i) - 3 (ii) $\frac{3}{7}$ (iii) 5 (iv) $\frac{1}{3}$ (v) $-\frac{2}{5}$ (vi) $-\frac{5}{8}$

b) (i) No (ii) No

c) (i) 0.3676 (ii) ± 1.775

4. a) (i) $\frac{5}{16}$ (ii) $\frac{13}{20}$ (iii) $\frac{133}{52}$ (iv) $\frac{87}{320}$

b) (i) $\frac{1}{1170}$ (ii) $-\frac{2}{195}$ (iii) $\sqrt[3]{\left(\frac{200}{217}\right)}$

5. (i) a) 34 b) 67 c) 94 d) - 3 e) - 1

(ii) a) ± 4.12 b) ± 2.34 c) ± 3.20 6. -1.84, 3.04 7. b) 169

SIGMA NOTATION

NOTES:

(1) The symbol \sum means 'summation'.

(2) Thus $\sum_1^5 r = 1 + 2 + 3 + 4 + 5 = 15$.

(3) $\sum_1^5 r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 1 + 4 + 9 + 16 + 25 = 55$.

(4) $\sum_1^5 r^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 1 + 8 + 27 + 64 + 125 = 225$.

(5) $\sum_3^7 (8r + 9) = (24+9) + (32+9) + (40+9) + (48+9) + (56+9)$
 $= 33 + 41 + 49 + 57 + 65 = 245$.

(6) $\sum_{r=5}^{16} r = \sum_{r=1}^{16} r - \sum_{r=1}^4 r$.

STANDARD SUMMATION FORMULAE

(7) $\sum_{r=1}^n r = \frac{n}{2}(n+1)$,

(8) $\sum_{r=1}^n r^2 = \frac{n}{6}(n+1)(2n+1)$,

(9) $\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$.