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Practice Questions with answers in Pure Advanced Level Mathematics Book 1

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CHAPTER 1

REASONING AND LOGIC

TRUTH TABLES

NOTES:

(1) The symbol $\wedge$ means 'and'.

(2) The symbol $\vee$ means 'or'.

(3) The symbol ~ means 'not', **this is negation.**

(4) A **proposition** is a sentence which assumes the truth value T or false, F.

Sometimes the number 1 is used for TRUE and 0 is used for FALSE.

(5) If p and q are **propositions** then $p \wedge q$ means 'p and q'.

(6) If p and q are **propositions** then $p \vee q$ means 'p or q'.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Complete the following truth table.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee q$	$p \vee \sim q$
T	T	T	F	F	F	T	
T	F	F	T	F	T	F	
F	T	F	T	T	F	T	
F	F	F	T	T	T	T	

Complete the following truth table.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \vee (q \vee r)$	$q \wedge r$	$p \vee (q \wedge r)$	$\sim(q \vee r)$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

NOTES:

(1) If x and y are propositions and at least one is FALSE, then $(x \wedge y)$ is FALSE.

(2) If x and y are propositions and at least one is TRUE, then $(x \vee y)$ is TRUE.

(3) $z \vee (x \wedge y) = (z \vee x) \wedge (z \vee y)$.

(4) $z \wedge (x \vee y) = (z \wedge x) \vee (z \wedge y)$.

In the table below, verify that columns (5) and (8) are equivalent.

$z \vee (x \wedge y) = (z \vee x) \wedge (z \vee y)$ [The two statements are logically equivalent.]

x	y	z	$x \wedge y$	$z \vee (x \wedge y)$	$z \vee x$	$z \vee y$	$(z \vee x) \wedge (z \vee y)$
T	T	T					
T	T	F					
T	F	T					
F	T	T					
T	F	F					
F	T	F					
F	F	T					
F	F	F					

Complete the table below to verify that **$z \wedge (x \vee y)$ is logically equivalent to**

$(z \wedge x) \vee (z \wedge y)$.

$z \wedge (x \vee y) = (z \wedge x) \vee (z \wedge y)$

x	y	z	$x \vee y$	$z \wedge (x \vee y)$	$z \wedge x$	$z \wedge y$	$(z \wedge x) \vee (z \wedge y)$
T	T	T					
T	T	F					
T	F	T					
F	T	T					
T	F	F					
F	T	F					
F	F	T					
F	F	F					

NOTES:

(1) Let p = the hypothesis and q = the conclusion.

***p* → *q* means *p* implies *q*.**

(2) If the statements p and q are both true, then p implies q or $(p \rightarrow q)$ is true.

(3) If the statements p and q are both false, then p implies q or $(p \rightarrow q)$ is true.

(4) If p is true and q is false then p implies q or $(p \rightarrow q)$ is false.

(5) If p is false and q is true then p implies q or $(p \rightarrow q)$ is true. That is, it is true that the false hypothesis is false.

In other words,

Column 1	Column 2	Column 3	Column 4
p	q	$p \rightarrow q$	Column 3 states that
T	T	T	It is **true** that the true hypothesis is true.
T	F	F	It is **false** that the true hypothesis is false.
F	T	T	It is **true** that the false hypothesis is true.
F	F	T	It is **true** that the false hypothesis is false.

Complete the table below.

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	T			
T	F	F			
F	T	T			
F	F	T			

Complete the table below.

p	r	$\sim p$	$\sim r$	$\sim p \vee \sim r$	$\sim p \wedge \sim r$	$\sim(p \vee r)$	$\sim(p \wedge r)$
T	T						
T	F						
F	T						
F	F						

NOTE THAT

(1) $\sim (p \vee r) = \sim p \wedge \sim r$ and (2) $\sim (p \wedge r) = \sim p \vee \sim r$.

CHAPTER 2

THE REAL NUMBER SYSTEM,

SIGMA NOTATION,

MATHEMATICAL INDUCTION

THE REAL NUMBER SYSTEM

NOTES:

Let $a, b, c \in R$,

(1) $a + b = c$ for all $a, b, c \in R$. The set of real numbers is ***closed*** under addition.

(2) $a - b = c$ for all $a, b, c \in R$. The set of real numbers is ***closed*** under subtraction.

(3) $ab = c$ for all $a, b, c \in R$. The set of real numbers is ***closed*** under multiplication.

(4) $\frac{a}{b}$ ***is not necessarily an integer***.

Thus, the set of real numbers is ***closed*** under division.

(5) $a + b = b + a$. ***Addition is commutative***.

(6) $a + (b + c) = (a + b) + c$. ***Addition is associative***.

(7) $a - b \neq b - a$. ***Subtraction is not commutative***.

(8) $a - (b - c) \neq (a - b) - c$. ***Subtraction is not associative***.

(9) $ab = ba$. ***Multiplication is commutative***.

(10) $(ab)c = a(bc)$. ***Multiplication is associative***.

(11) $a + 0 = a$. ***The identity element under addition is ZERO***.

(12) $a + (-a) = 0$. ***The inverse element under addition is (- a)***.

(13) $a \times 1 = a$. ***The identity element under multiplication is ONE***.

(14) $a \times \left(\frac{1}{a}\right) = 1$. ***The inverse element under multiplication*** $\left(\frac{1}{a}\right)$.

BINARY OPERATIONS

EXERCISE 2A

1. a) Given that x * y = $2x$ - y, calculate (i) 2 * 3, (ii) 3 * 2, (iii) (3 * 2) * (- 4)

(iv) 8 * (2 * 3).

b) Solve the equation (i) f * 9 = 10, (ii) r * (- 8) = 12.

2. a) The operation * is defined by x * y = $3x + 4y$. Calculate (i) 5 * 6,

(ii) 6 * 5, (iii) 3 * (6 * 5), (iv) (3 * 6) * 5, (v) (- 8) * (-9).

b) Solve, giving the ***exact*** solution(s) of each of the following equations.

(i) y * 4 = 14, (ii) x^2 * 2 = 50, (iii) k^7 * (- 6) = 40.

3. a) If $r * t = \frac{r+t}{r-t}$, calculate the ***exact*** value of each of the following.

(i) 3 * 6, (ii) 5 * (- 2), (iii) 6 * 4, (iv) (- 10) * (5), (v) (3 * 6) * 7, (vi) 3 * (6 * 7).

b) (i) Is * associative? (ii) Show that * is not commutative.

c) Solve each of the following equations correct to 4 significant figures.

(i) $(17x) * 5 = 9$, (ii) $3x^2 * 8 = 12$.

4. a) Given that $f * g = \dfrac{f+g}{4f}$, calculate each of the following in ***fractional form***.

(i) 8 * 2, (ii) 5 * 8, (iii) (5 * 8) * 6, (iv) 5 * (8 * 6).

b) Solve, giving the ***exact*** solution, of each of the following equations.

(i) $6x * 1 = 49$, (ii) $5x * (-2) = 10$, (iii) $7x^3 * 200 = 8$.

5. (i) Given that $p * r = p^2 - r$, calculate a) 6 * 2, b) 8 * (-3), c) (- 10) * 6,

d) (2 *3) * 4, e) 2 * (3 * 4).

(ii) Solve, giving the solutions correct to 2 decimal places, the equation a) $x * 7 = 10$,

b) $x^5 * 8 = 4934$, c) $3x^2 * (-17) = 960$.

6. The operation * is defined as $x * y = 5x^2 - 6x - 7y^2$. Solve the equation $x * 2 = 0$,

giving your answers correct to 2 decimal places.

7. The operation # is defined as $p \# r = p^2 + r^2 - 9pr$.

a) Prove that # is ***commutative***.

b) Calculate 5 # (- 3).

ANSWERS TO EXERCISE 2A

1 a) (i) 1 (ii) 4 (iii) 12 (iv) 15

b) (i) $f = 9.5$ (ii) $r = 2$

2. a) (i) 39 (ii) 38 (iii) 161 (iv) 119 (v) - 60

b) (i) $-\frac{2}{3}$ (ii) $\pm\sqrt{14}$ (iii) $\sqrt[7]{\left(\frac{64}{3}\right)}$

3. a) (i) - 3 (ii) $\frac{3}{7}$ (iii) 5 (iv) $\frac{1}{3}$ (v) $-\frac{2}{5}$ (vi) $-\frac{5}{8}$

b) (i) No (ii) No

c) (i) 0.3676 (ii) ± 1.775

4. a) (i) $\frac{5}{16}$ (ii) $\frac{13}{20}$ (iii) $\frac{133}{52}$ (iv) $\frac{87}{320}$

b) (i) $\frac{1}{1170}$ (ii) $-\frac{2}{195}$ (iii) $\sqrt[3]{\left(\frac{200}{217}\right)}$

5. (i) a) 34 b) 67 c) 94 d) - 3 e) – 1

(ii) a) ± 4.12 b) $\pm$ 2.34 c) $\pm$ 3.20 6. -1.84, 3.04 7. b) 169

SIGMA NOTATION

NOTES:

(1) The symbol $\sum$ means 'summation'.

(2) Thus $\sum_{1}^{5} r = 1 + 2 + 3 + 4 + 5 = 15.$

(3) $\sum_{1}^{5} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 1 + 4 + 9 + 16 + 25 = 55.$

(4) $\sum_{1}^{5} r^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 1 + 8 + 27 + 64 + 125 = 225.$

(5) $\sum_{3}^{7} (8r + 9) = (24+9) + (32+9) + (40+9) + (48+9) + (56+9)$

$= 33 + 41 + 49 + 57 + 65 = 245.$

(6) $\sum_{r=5}^{16} r = \sum_{r=1}^{16} r - \sum_{r=1}^{4} r.$

STANDARD SUMMATION FORMULAE

(7) $\sum_{r=1}^{n} r = \frac{n}{2}(n+1),$

(8) $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1),$

(9) $\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2.$